not a jump, at $(p/p_0) = 0$ For example, with $\nu = 0.3$ and $(l/R) = 4$, one obtains the following:

1) For $(R/t) = 500$ at $(\sigma/\sigma_0) = 0.01$, the minimum φ_L occurs at $m = 3$ and not at $m = 1$

2) For $(R/t) = 200$ at $(\sigma/\sigma_0) = 0.02$, the minimum φ_L occurs at $m = 2$ and not at $m = 1$

Experimental evidence (although not directly applicable, since it refers to actual shells whose buckling behavior deviates from that predicted by linear theory) also confirms that $m > 1$ for some cases of combined axial compression and lateral pressure (see, for example, Ref 2)

References

[1] Sharman, P W, "A theoretical interaction equation for the buckling of circular shells under axial compression and external pressure" J Aerospace Sci **29**, 878–879 (1962)

[2] Weingarten, V I, Morgan, E J and Seide, P, 'Final report on the development of design criteria for elastic stability of thin shell structures Space Technology Labs, Los Angeles, TR 60 0000 1945, p 175 (December 1960)

Reply by Author to J Singer

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THE author is grateful to Dr Singer for pointing out the slight error in his statement

The main utility of an interaction equation is, of course, to provide a rapid means of design rather than an exact analysis An advantage of formulating such an equation in terms of "reserve factors" is that the denominators may be calculated from theoretical or empirical formulas, although such a mixture of theory and experiment is not really satisfactory However, in the absence of close agreement with theory and experiment, the procedure may be acceptable for design estimates

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Comments on "Mach Number Independence of the Conical Shock Pressure Coefficient"

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THE results presented in this note[1] may be compared with those of a recent technical note[2] which also presented an approximate equation for shock wave angle as a function of cone angle and Mach number In addition, the effect of specific heat ratio was included, and an equation for the surface pressure coefficient was developed Comparisons with exact results were shown for Mach number from 1 05 to 20 0, cone angle from 0 to the detachment value, and specific heat ratio from 1 1 to 1 8

Simpler equations were also presented [Eqs (12) and (13)][2] for the case when the sine of the cone angle was less than 85% of the detachment value In the nomenclature of the present note,[1] these equations would be

$$\sin\theta_w = \left[\frac{\gamma+1}{2}\sin^2\theta + \frac{1}{M_\infty^2}\right]^{1/2} \tag{1}$$

$$C_P = \left[\frac{\gamma+7}{4} - \left(\frac{\gamma-1}{4}\right)^2 + \frac{6}{M_\infty^6} + \frac{M_\infty^2 - 1}{M_\infty^4 \sin\theta}\right]\sin^2\theta \tag{2}$$

Equation (10) of the present note, with θ in radians, is

$$\sin\theta_w = \left[\theta^{1.87} + \frac{1}{M_\infty^2}\right]^{1/2} \tag{3}$$

In the limit of Newtonian flow ($M_\infty \to \infty$, $\gamma \to 1$), Eq (3) becomes

$$\sin\theta_w = \theta_c^{0.935} \tag{4}$$

In the same limit, Eq (1) becomes

$$\sin\theta_w = \sin\theta \tag{5}$$

Since in Newtonian flow $\theta_w = \theta$, it is clear that the form of the approximation in the present note is not valid in the limit, even though the numerical values for air are reasonable In addition, the effect of specific heat ratio on shock angle is not considered

References

[1] Zumwalt, G W and Tang, H H, 'Mach number independence of the conical shock pressure coefficient," AIAA J **1**, 2389–2391 (1963)

[2] Simon, W E and Walter, L A, 'Approximations for supersonic flow over cones," AIAA J **1**, 1696–1697 (1963)

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Reply by Author to W E Simon

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THE technical note of Simon and Walter[1] was published just after the final form of the paper being discussed[2] was submitted, so there was no opportunity for prepublication comparison of the two works As Simon points out, they have indeed succeeded in including the effect of the specific heat ratio in their conical shock approximations, and their curves then draw attention to the insensitivity of conical shock angle and surface pressure coefficient to γ values for the range applicable to perfect gas analysis They have obtained very good agreement for the $\gamma = 1.405$ value and for other γ values at the one cone angle of 20° No doubt they experienced the same difficulty as we in checking results for other γ values because of the lack of available published cone-flow solutions

In answer to Simon's criticism, it should be pointed out that our principal purpose was to call attention to the conical shock-wave pressure coefficient's strange behavior Secondarily, as a suggested use of this fact, an approximation for the conical wave angle was developed The resulting equation was similar in form to that of Simon and Walter, and it is not obvious to me that their equation is more simple Our equation fails to agree with the Newtonian limit, it is true However, Simon and Walter's equation (13) fails in exactly the same way, giving values almost identical to ours, unless the γ value is arbitrarily changed to unity The

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much-used Kopal-computed values, as plotted in Ref 3, share this limitation with us

References

[1] Simon, W E and Walter, L A ' Approximations for supersonic flow over cones," AIAA J 1, 1696–1698 (1963)

[2] Zumwalt, G W and Tang, H H "Mach number independence of the conical shock pressure coefficient," AIAA J 1, 2389–2391 (1963)

[3] "Equations, tables, and charts for compressible flow," Ames Res Lab , NACA Rept 1135, p 48 (1953)

Reply by Author to R E Lavender

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An analytical procedure was outlined in Ref 1 for describing the touchdown dynamics of a lunar spacecraft The motion of the vehicle was defined by the classical Eulerian equations of rigid body motion

The application of external forces to the vehicle is determined by two controls incorporated in the digital computer program These two conditions require that a vehicle leg tip be in contact with the lunar surface and also have a resultant lateral or (positive) vertical velocity The velocities are referred to the body frame of reference When both conditions hold, forces equal to the crushing forces of the inelastic energy absorbers are applied to the leg tips The motion of each crushed leg tip is traced, and its relative position to the lunar surface is determined The deformation function described in Ref 1 is used to ascertain this relationship for each leg A positive or zero value of the function indicates surface contact for the respective vehicle legs Any implication of leg liftoff is determined from the actual trace of the leg-tip position and not from leg crushing rates However, the amount of permanent deformation of the vertical energy absorber can be determined from the deformation function

The activation of the lateral and vertical energy absorbers occurs independently For example, if the lateral leg-tip velocity is reduced to zero, the lateral force applied to that leg is zero This does not restrict activation of the vertical energy absorber if the leg tip still has vertical velocity When both vertical and lateral tip velocities vanish, the forces are removed and the vehicle undergoes rigid body motion provided the other legs are not in the process of crushing Realistically, since a step-by-step integration procedure is used, the velocities will most likely not vanish identically The small residual velocities that exist will induce external forces However, the force applications will be such as to approximate the true leg positions within the requirements dictated by the surface friction coefficient The end result will be that the time average of load applications will approximate the actual load history The smaller the time increments considered the better the approximation

As Lavender has correctly pointed out, forces of reduced magnitude are actually applied to the vehicle when the tip velocities vanish The writer was well aware of this condition when the problem was formulated three years ago It was decided at that time that for the inelastic energy absorbers under consideration the magnitude of the total impulse applied to the vehicle because of these reduced forces was small compared to the crushing forces Correspondingly, the stability profile would not be seriously affected The basic program has since been modified by W D Brayton of the Space & Information Division to include the effects of reduced impact forces In cases run by Brayton, it was found that these forces are of little consequence and do not critically affect the stability profile The modified program has the added capability of considering the coefficient of friction, multistage landing systems, rotating landing gears, elasticity in the landing legs, and other refinements

The question of the validity of the outlined assumptions can only be answered by correlation with empirical evidence The outlined analytical procedure has been compared with the results of three independent drop test series to obtain this verification The first series of tests connected with the Surveyor program were outlined in Ref 1 Since publication of the paper, comparison has also been made with two other recent tests Figure 1 indicates the results of a series of 36 drop tests performed at the Space & Information Systems Division Development Laboratory The stability

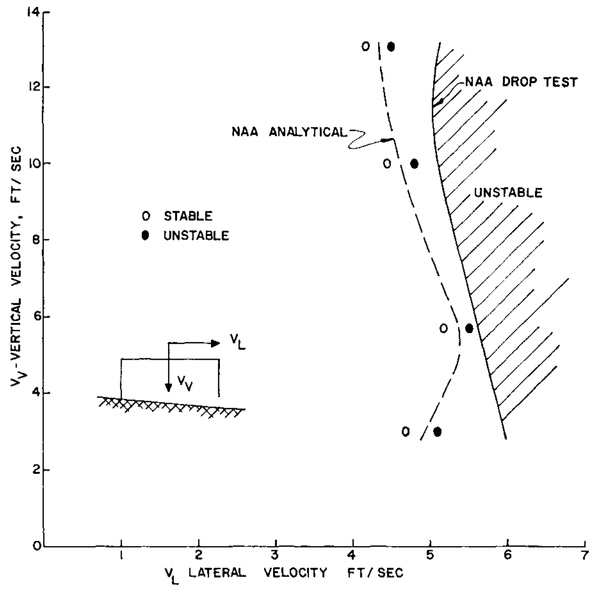

Fig 1 Lunar landing simulation test, North American Aviation Inc (Model data: $R_1 = 1\ 55$ ft; $R_2 = 1\ 342$ ft; $R_3 = 0\ 775$ ft; $DCG = 1\ 555$ ft; $nF_V/m = 578$ ft/sec²; $I_y/m = 0.345$ ft²; $I_z/m = 0\ 345$ ft²; coefficient of friction = 0 5; $g = 32\ 2$ ft/sec²; $A = 5$; $B = 0°$; $\beta = 270°$)

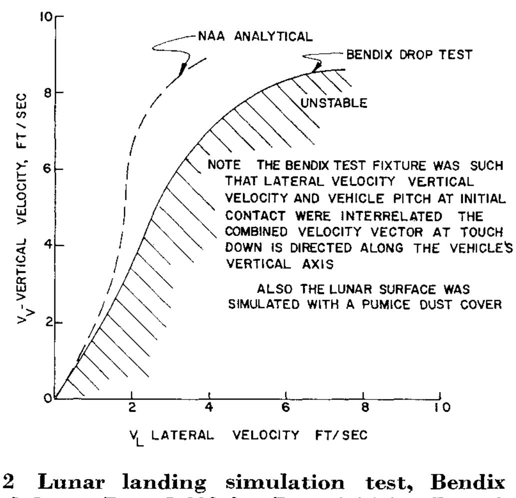

Fig 2 Lunar landing simulation test, Bendix Corp (Model data: $R_1 = 1\ 212$ ft; $R_2 = 1\ 05$ ft; $R_3 = 0\ 607$ ft; $DCG = 1\ 506$ ft; $nF_V/m = 462$ ft/sec²; $I_x/m = 0\ 920$ ft²; $I_y/m = 0\ 920$ ft²; $I\ /m = 0\ 833$ ft²; coefficient of friction = 0 5; $g = 32\ 2$ ft/sec²; $A = 5°$; $B = 0°$; $\beta = 270°$)

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